

# Correlation between muonic levels and nuclear structure in muonic atoms

J. M. Dong,<sup>1,2,3,4,5</sup> W. Zuo,<sup>1,2,4</sup> H. F. Zhang,<sup>4</sup> W. Scheid,<sup>6</sup> J. Z. Gu,<sup>5</sup> and Y. Z. Wang<sup>5</sup>

<sup>1</sup>Research Center for Nuclear Science and Technology,

Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

<sup>2</sup>Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

<sup>3</sup>Graduate University of Chinese Academy of Sciences, Beijing 100049, China

<sup>4</sup>School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

<sup>5</sup>China Institute of Atomic Energy, P. O. Box 275(18), Beijing 102413, China

<sup>6</sup>Institute for Theoretical Physics, Justus-Liebig-University, D-35392 Giessen, Germany

A method that deals with the nucleons and the muon unitedly is employed to investigate the muonic lead, with which the correlation between the muon and nucleus can be studied distinctly. A “kink” appears in the muonic isotope shift at a neutron magic number where the nuclear shell structure plays a key role. This behavior may have very important implications for the experimentally probing the shell structure of the nuclei far away from the  $\beta$ -stable line. We investigate the variations of the nuclear structure due to the interaction with the muon in the muonic atom and find that the nuclear structure remains basically unaltered. Therefore, the muon is a clean and reliable probe for studying the nuclear structure. In addition, a correction that the muon-induced slight change in the proton density distribution in turn shifts the muonic levels is investigated. This correction to muonic level is as important as the Lamb shift and high order vacuum polarization correction, but is larger than anomalous magnetic moment and electron shielding correction.

PACS numbers: 36.10.Dr, 27.80.+w, 21.60.Jz, 23.40.-s

Keywords: muonic atoms, relativistic mean field, nuclear shell structure, isotope shift, muonic spectrum

A muonic atom is an atom in which one of the electrons has been replaced by a negatively charged muon. Because of the very large mass of a muon compared with a electron and the correspondingly small Bohr radius, the muonic wave function has a large overlap with the nucleus. X-ray transition energies in muonic atoms are strongly affected by the size of the nuclei, and can be used efficiently to determine the nuclear charge distribution [1, 2]. A detailed introduction about muonic atoms can be found in Ref. [3]. Moreover, the muonic atom tends to play an important role in investigation on other subjects. A recent study demonstrated that muonic atoms in strong laser fields can be used to dynamically gain structure information on nuclear ground states [4]. Masafumi Koike *et al.* proposed a new process of  $\mu^- e^- \rightarrow e^- e^-$  in a muonic atom for a quest of charged lepton flavor violation (This violation is known to be one of the important rare processes to search for new physics beyond the standard model) in consideration of the fact that this process in a muonic atom has various significant advantages [5]. An attractive means to improve the accuracy in the measurement of proton root-mean square radius is provided by muonic hydrogen [6]. In addition, some potential synergies of combining muons with radioactive nuclei may become a new tool to be used at future RIB facilities, as suggested in Ref. [7]. Therefore, investigation on muonic atom is meaningful and not limited to atomic physics.

We extend the relativistic mean field (RMF) approach in this Letter to include the negatively charged muon, and then the correlation between the muon and nucleus, namely the effects of the nuclear structure on the muonic spectra as well as the influence of the muon on the nuclear structure, can be investigated distinctly. Within this cor-

relation, we may find some new approach to probe the nuclear structure. Only one muon captured by the nucleus is discussed here. Because the motion of the bound muon is relativistic for high- $Z$  atoms, it is necessary to treat it in relativistic framework. Nowadays the RMF theory has become a standard tool in low energy nuclear structure [8–10] and the interacting Lagrangian density taking into account the muonic field is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\nu \partial_\nu - M)\psi + \frac{1}{2}\partial_\nu \sigma \partial^\nu \sigma \\ & - (\frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{3}g_2 \sigma^3 + \frac{1}{4}g_3 \sigma^4) - g_\sigma \bar{\psi}\psi \sigma \\ & - \frac{1}{4}\Omega_{\nu\lambda}\Omega^{\nu\lambda} + \frac{1}{2}m_\omega^2 \omega_\nu \omega^\nu - g_\omega \bar{\psi}\gamma^\nu \psi \omega_\nu \\ & - \frac{1}{4}R_{\nu\lambda} \cdot R^{\nu\lambda} + \frac{1}{2}m_\rho^2 \rho_\nu \cdot \rho^\nu - g_\rho \bar{\psi}\gamma^\nu \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\nu \psi \\ & - \frac{1}{4}F_{\nu\lambda}F^{\nu\lambda} - e\bar{\psi}\gamma^\nu \frac{1+\tau_3}{2}\psi A_\nu \\ & + \bar{\psi}_\mu(i\gamma^\nu \partial_\nu - m_\mu)\psi_\mu - (-e)\bar{\psi}_\mu\gamma^\nu \psi_\mu A'_\nu. \end{aligned} \quad (1)$$

$M$ ,  $m_\sigma$ ,  $m_\omega$  and  $m_\rho$  are the nucleon-, the  $\sigma$ -, the  $\omega$ - and the  $\rho$ -meson masses, respectively. The muon mass of  $m_\mu = 105.6583668$  MeV is taken from Ref. [11]. The nucleon field  $\psi$  interacts with the  $\sigma, \omega, \rho$  meson fields  $\sigma, \omega_\nu, \rho_\nu$  and with the photon field  $A_\nu$ . The muonic field  $\psi_\mu$  interacts with the photon field  $A'_\nu$  and  $A'_\nu$  excludes the photon field produced by the muon itself. The field tensors for the vector meson are given as  $\Omega_{\nu\lambda} = \partial_\nu \omega_\lambda - \partial_\lambda \omega_\nu$  and by similar expression for  $\rho$  meson and the photon. The self-coupling terms with coupling constants  $g_2$  and  $g_3$  for the  $\sigma$  meson are introduced which turned out to be crucial [12]. Since the muon couples only electromagnetically to nucleons, no additional parameters need to be

introduced, and no readjustment of the present parameters is needed. Varying the effective Lagrangian, one can obtain the Dirac equation for nucleons and muons, and the Klein-Gordon equations for mesons. The calculations are performed in coordinate space using a mesh size of 0.01 fm and different box sizes for different muonic states, and the pairing correlations are accounted in the BCS formalism with an energy gap  $\Delta$  obtained from the observed odd even mass differences for an open shell and  $\Delta = 0$  for a closed shell. The NL3 and NLSH parameter sets are employed here. The NL3 parameter set has been used with enormous success in the description of a variety of ground-state properties of spherical, deformed and exotic nuclei [13, 14], and NLSH is also a successful parameter set [15]. The nucleons and muon are treated in the unified framework without any adjustable parameter and the finite size effect of the nucleus is automatically included so that both the muonic atom structure and nuclear structure can be investigated simultaneously. In contrast to earlier works about the muonic atoms based on Migdal theory [16, 17], the present method is relativistic. Also, our approach is more microscopic compared with the method that directly solves the Dirac equation using the two or three-parameter Fermi-type distribution of the nuclear charge [18, 19]. In fact, it was a common practice to fit the parameter sets for this nuclear charge distributions with the help of the experimental muonic levels [18]. In the RMF theory, the nucleons are treated as point particles, which is a drawback of RMF theory. As a matter of fact, the finite size effects of the nucleons cause some influence on the nuclear properties. The muonic spectrum is affected by the finite size effects of the protons because the distributions of charges differ slightly from those of the protons. But in the self-consistent method, the proton density distributions are used as sources rather than the charge distributions. In our calculations, the Coulomb potential that the muon feels excludes the potential produced by the muon itself. In other words, the muon only interacts with the electrostatic potential generated by the protons. In fact, the mean field that the muon feels is a central potential field for a spherical nucleus, and the corresponding muonic energy  $E_{\mu 0}$  is obtained by applying the RMF approach discussed above.

Once the Dirac wave functions of the muon are obtained with the RMF method, the lowest order vacuum polarization correction  $\Delta\varepsilon_\mu$ , as shown in Fig. 1, can be calculated additionally. The details on this lowest order vacuum polarization correction can be found in Ref. [19]. The ultimate calculated muonic levels  $E_\mu = E_{\mu 0} + \Delta\varepsilon_\mu$  in the muon-Pb systems are presented in Table I. As can be seen, the results of the RMF method taking into account lowest order vacuum polarization corrections agree with the experimental data suggesting the effectiveness of the RMF to describe the muonic atoms. The lowest order vacuum polarization correction is relatively large for low energy levels, but the correction become smaller for high energy levels. The nuclear polarization that

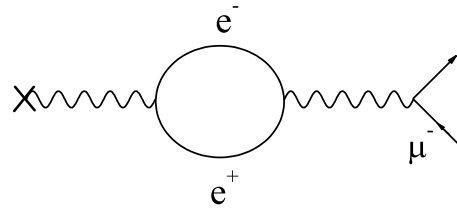


FIG. 1: Lowest order vacuum polarization diagram.

arises the muon-nucleus electrostatic interaction excites the nucleus into virtual excited states, is partly taken into account due to the self-consistent calculations, as will be shown below. Other corrections such as high order vacuum polarization correction, Lamb shift, anomalous magnetic moment, electron shielding and static hyperfine structure interaction, contribute little to the binding energy of the muon atoms compared with the lowest order vacuum polarization correction [22] and these effects are inessential for the following discussions, so they are not considered here. Note that the uncertainties of muonic levels especially  $1s_{1/2}$  levels in heavy nuclei are mainly ascribed to the nuclear structure, i.e., the proton density distribution. The agreement between the calculated and experimental muonic levels indicates the reliability of the proton distribution from the RMF calculations. The  $1s_{1/2}$  levels with NL3 are in better agreement with the experimental data than those obtained with NLSH, possibly because that NL3 can provide a more accurate proton density distribution over the parameter set NLSH. The parametrization NL3 is more excellent because it cures some deficiencies of the NLSH to some extent, as suggested in Ref. [13]. In addition, the mean speed of the muon in  $1s_{1/2}$  orbit for the muon- $^{208}\text{Pb}$  system is  $v = 0.31c$  according to our calculations, indicating the importance of relativistic effect. Thus the treating of the muon and nucleons in a unified relativistic framework is necessary.

In Fig. 2(a), the nuclear isotope shifts  $\langle r^2 \rangle_c - \langle r^2 \rangle_c(^{208}\text{Pb})$  for the Pb isotope chain are plotted as a function of neutron number, taking the nucleus  $^{208}\text{Pb}$  as a reference. With the increase of the neutron number  $N$ , the charge radius squared changes only slightly until it reaches the magic neutron number  $N = 126$ , but it increases more rapidly with the neutron number  $N$  goes beyond  $N = 126$ . In other words, a “kink” in the nuclear isotope shift is found at  $N = 126$ . We would like to mention that the non-relativistic calculations have not succeeded in offering this fact which originates from the nuclear shell effect. The muonic isotope shift (isotope shift of atomic spectrum) which is the shift of the muonic

TABLE I: The muonic spectrum obtained with the RMF method including the lowest order vacuum polarization correction for the muonic  $^{204,206,208}\text{Pb}$ . The mean field part  $E_{\mu 0}$  and the lowest order vacuum polarization correction  $\Delta\varepsilon_{\mu}$  are listed and the muonic levels are given by  $E_{\mu} = E_{\mu 0} + \Delta\varepsilon_{\mu}$ . The experimental data [18] have been listed for comparison. All energies are in units of keV.

A	orbit	$E_{\mu 0}(\text{NL3})$	$\Delta\varepsilon_{\mu}(\text{NL3})$	$E_{\mu 0}(\text{NL3H})$	$\Delta\varepsilon_{\mu}(\text{NL3H})$	$E_{\mu}(\text{NL3})$	$E_{\mu}(\text{NL3H})$	$E_{\mu}(\text{Expt.})$
204	$1s_{1/2}$	-10549.41	-66.35	-10561.24	-66.47	-10615.76	-10627.72	$-10614.88 \pm 0.41$
204	$2p_{1/2}$	-4781.17	-31.26	-4783.41	-31.30	-4812.43	-4814.71	$-4818.62 \pm 0.08$
204	$2p_{3/2}$	-4597.97	-28.70	-4599.67	-28.73	-4626.67	-4628.40	$-4632.72 \pm 0.06$
204	$2s_{1/2}$	-3583.87	-18.26	-3585.90	-18.28	-3602.13	-3604.18	$-3604.14 \pm 0.26$
204	$3d_{3/2}$	-2161.81	-9.40	-2161.86	-9.40	-2171.21	-2171.26	$-2173.03 \pm 0.05$
204	$3p_{3/2}$	-2082.71	-9.20	-2083.28	-9.21	-2091.90	-2092.49	$-2092.41 \pm 0.33$
206	$1s_{1/2}$	-10540.54	-66.25	-10551.51	-66.37	-10606.80	-10617.87	$-10604.34 \pm 0.41$
206	$2p_{1/2}$	-4779.84	-31.23	-4781.97	-31.27	-4811.07	-4813.24	$-4817.19 \pm 0.06$
206	$2p_{3/2}$	-4597.03	-28.68	-4598.65	-28.71	-4625.71	-4627.37	$-4631.50 \pm 0.05$
206	$2s_{1/2}$	-3582.23	-18.24	-3584.08	-18.26	-3600.47	-3602.34	$-3601.54 \pm 0.34$
206	$3d_{3/2}$	-2161.78	-9.40	-2161.84	-9.40	-2171.18	-2171.23	$-2173.02 \pm 0.05$
206	$3p_{3/2}$	-2082.38	-9.19	-2082.93	-9.20	-2091.57	-2092.13	$-2092.01 \pm 0.39$
208	$1s_{1/2}$	-10532.13	-66.16	-10542.23	-66.26	-10598.29	-10608.49	$-10593.13 \pm 0.41$
208	$2p_{1/2}$	-4778.56	-31.21	-4780.59	-31.25	-4809.77	-4811.83	$-4815.14 \pm 0.06$
208	$2p_{3/2}$	-4596.13	-28.66	-4597.68	-28.69	-4624.79	-4626.37	$-4630.25 \pm 0.05$
208	$2s_{1/2}$	-3580.66	-18.22	-3582.35	-18.24	-3598.88	-3600.58	$-3599.75 \pm 0.17$
208	$3d_{3/2}$	-2161.76	-9.40	-2161.81	-9.40	-2171.15	-2171.21	$-2173.01 \pm 0.05$
208	$3p_{3/2}$	-2082.07	-9.19	-2082.59	-9.20	-2091.25	-2091.79	$-2092.20 \pm 0.32$

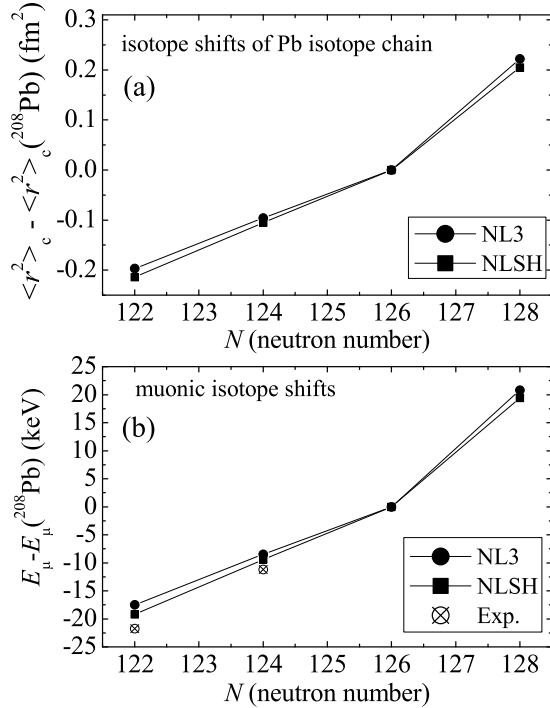


FIG. 2: (a) Nuclear isotope shifts of the Pb isotope chain and (b) muonic isotope shifts of  $1s_{1/2}$  state in muonic Pb. The  $^{208}\text{Pb}$  is taken as a reference nucleus and the experimental data are taken from Ref. [18].

energy with increasing neutron number, includes the field shift coming from the change of spacial distribution of the charges in the nucleus and mass shift originating from the change of the mass of the nucleus. For a heavy nucleus, the field shift is major and much larger than the mass shift. In heavy elements such as lead, the mass effect is negligible and the field effect roughly accounts for the observed shifts. Fig. 2(b) displays the muonic isotope shift taking the muonic energy in  $^{208}\text{Pb}$  as a reference. Hence computational errors along with various corrections cancel to a high degree. This muonic isotope shift can be attributed to the nuclear isotope shift. The larger the nuclear charge radius is, the wider the charge distributes since the total charge is invariant, and hence a high muonic energy. A “kink” in the muonic isotope shift is also found at  $N = 126$ , which stems from the neutron shell effect in the final analysis. For other isotope chains, the “kink” can be also found at magic number, and we do not discuss them in detail here. This is certainly a good news for the nuclear structure study. The “kink” in the muonic isotope shifts may be taken as a useful signal to experimentally probe the shell structure of the nuclei far away from the  $\beta$ -stable line for which novel nuclear structure effects may exist. This is one of the main conclusions we draw in this work. Strasser *et al.* has proposed the cold hydrogen film method to extend muonic atom spectroscopy to the use of nuclear beams including radioactive isotope beams to produce radioactive muonic atom in the future [20]. This would allow studies of unstable nuclei by means of the muonic X-ray method at facilities where both  $\mu^-$  and radioactive isotope beams would be available. The electron scattering is not going to be easy to be applied for exotic nuclei with very low beam intensities when one measures the

scattering cross sections to obtain the knowledge about the nuclear structure. But for muonic atoms, one measures the X-ray energies, which is much easier and X-ray energy can be measured with a high accuracy. For the other approach to probe the nuclear shell structure, the first excited  $2^+$  state, can be carried out using Coulomb excitation [21] for these exotic nuclei. Therefore, as two independent methods, the approaches of Coulomb excitation and muonic atom spectroscopy can complement each other.

TABLE II: The correction for muonic  $1s_{1/2}$  orbit due to the muon-induced slight change of the proton density distributions in muonic Pb.

Nucleus	NL3	NLSH
$^{204}\text{Pb}$	-1.5 keV	-1.2 keV
$^{206}\text{Pb}$	-1.5 keV	-1.2 keV
$^{208}\text{Pb}$	-1.5 keV	-1.2 keV
$^{210}\text{Pb}$	-1.5 keV	-1.2 keV

Apart from obtaining information on the muonic spectrum, the influence of the muon on nuclear structure can also be investigated. It is found that the nuclear structure of the Pb isotope remains basically unchanged in the presence of the muon in  $1s_{1/2}$  orbit. The single particle level spacing of neutrons and protons is altered by only several or some dozen keV. The nucleon densities are reduced by about 0.1% at the edge of the nucleus ( $\sim 7$  fm) but enhanced in the interior of nucleus by about 0.1%. In other words, the protons and neutrons move quite slightly towards the core as a result of the bound muon, and the nuclear rms radii are reduced only by about 0.02%. For the calculation of these changes in value, the systematic computational errors here could be canceled to a large extent, so that the results are reliable. The bound muon in other orbits affect the nuclear structure much more weakly than that in the  $1s_{1/2}$  orbit. The muon thus can be taken as a clean and reliable probe to extract information on nuclear structure since this probe can hardly change the nuclear structure. It is reliable to yield information on nuclear charge distributions and rms radii through muonic X-ray in experiments. That is to say, the previous experiments on the nuclear charge distributions and charge rms radii within the muonic atoms were theoretically confirmed to be reasonable and reliable. Moreover, the fact that the nuclear structure is only slightly changed by the muon implies that the linear response theory used in Ref. [16, 17] is an excellent approximation. Although the change of the proton density distribution due to the bound muon is quite slight, it in turn lowers the muonic levels especially for  $1s_{1/2}$  orbit, which is a part of nuclear polarization since only virtual excitation of  $J^\pi = 0^+$  states is taken into account here. This correction for muonic Pb is presented in Table II, which is more important than the anomalous magnetic moment and electron shielding, and can be compared to

Lamb shift and high order vacuum polarization corrections (For the  $1s_{1/2}$  orbit in muonic  $^{206}\text{Pb}$ , the corrections to muonic energy levels from anomalous magnetic moment, Lamb shift and high order vacuum polarization are 0.445 keV [22], 2.302 keV [22] and -1.75 keV [23], respectively. The electron shielding correction is even less, for example, -4.6 eV for the  $1s_{1/2}$  orbit in muonic  $^{209}\text{Bi}$  [24]). However, this effect tended to be neglected in many previous investigations which regarded the charge distribution as an invariant when a muon is captured. Yet it is not easy to be studied with the usual method but can be automatically included in our approach since the muon and nucleons are treated in a unified theoretical framework. Attention should be paid to this correction since the experimental measurements on muonic levels can achieve this precision.

In summary, the muonic atoms have been investigated in the framework of many-body theory. The main conclusions are summarized as follows: (1) The relativistic mean field theory for nuclei has been extended to investigate the muonic atoms of lead isotopes by taking the nucleons and a bound muon as a system and the theoretical muonic levels agree with the experimental data. Therefore, a direct link has been established between the nuclear physics and atomic physics. (2) The muonic isotopes shift stems from nuclear isotope shift for heavy nuclei. Just like the “kink” in nuclear isotope shift, a “kink” also appears in the muonic isotope shift at the neutron magic number. Perhaps it gives us a new approach to experimentally probe the shell structure of the nuclei far from the  $\beta$ -stable line via this “kink” in muonic isotope shift. (3) The muon-induced changes of nuclear structure are quite slight, which indicates the muon can be taken as a clean and reliable probe to extract information on nuclear structure, and theoretically confirms that the previous experiments on the nuclear charge distributions and charge rms radii within the muonic atoms were reasonable and reliable. (4) As a part of nuclear polarization, the correction that the muon-induced slight change in the proton density distribution in turn shifts the muonic levels is studied, which should be paid attention to because the contribution of it to muonic levels is more important than anomalous magnetic moment and electron shielding, and can be comparable to the Lamb shift and high order vacuum polarization correction.

J. M. Dong is thankful to Prof. Umberto Lombardo for helpful discussions. This work is supported by the National Natural Science Foundation of China (10875151, 10575119, 10975190, 10947109), the Major State Basic Research Developing Program of China under No. 2007CB815003 and 2007CB815004, the Knowledge Innovation Project (KJCX2-EW-N01) of Chinese Academy of Sciences, CAS/SAFEA International Partnership Program for Creative Research Teams (CXTD-J2005-1), the Funds for Creative Research Groups of China (Grant 11021504) and the financial support from DFG of Germany.

- 
- [1] John A. Wheeler, Phys. Rev. 92 (1953) 812 .
  - [2] G. Fricke, C. Bernhardt, K. Heilig, L. A. Schaller, L. Schellenberg, E. B. Shera, and C.W. de Jager, At. Data Nucl. Data Tables 60 (1995) 177.
  - [3] E. Borie and G. A. Rinker, Rev. Mod. Phys. 54 (1982) 67.
  - [4] A. Shahbaz, C. Müller, A. Staudt, T. J. Bürvenich, and C. H. Keitel, Phys. Rev. Lett. 98 (2007) 263901; A. Shahbaz, T. J. Bürvenich, and C. Müller, Phys. Rev. A 82 (2010) 013418.
  - [5] Masafumi Koike, Yoshitaka Kuno, Joe Sato and Masato Yamanaka, Phys. Rev. Lett. 105 (2010) 121601.
  - [6] Randolph Pohl *et al.*, nature (London) 466 (2010) 213.
  - [7] T. Nilsson, J. Äystö, K. Langanke, K. Riisager, G. Martinez-Pinedo, E. Kolbe, Nucl. Phys. A 746 (2004) 513c.
  - [8] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1 .
  - [9] Y. K. Gambhir, P. Ring, and A. Thimet, Ann. Phys. (NY) 198 (1990) 132.
  - [10] P. Ring, Prog. Part. Nucl. Phys. 37 (1996) 193.
  - [11] K. Nakamura *et al.* (Particle Data Group), J. Phys. G: Nucl. Part. Phys. 37 (2010) 075021.
  - [12] J. Boguta and A. R. Bodmer, Nucl. Phys. A 292 (1977) 413.
  - [13] G. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55 (1997) 540.
  - [14] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. 95 (2005) 122501.
  - [15] M. M. Sharma, M. A. Nagarajan, P. Ring, Phys. Lett. B 312 (1993) 377.
  - [16] J. Meyer, P. Ring and J. Speth, Phys. Rev. A 7 (1973) 1803.
  - [17] J. Meyer and J. Speth, Nucl. Phys. A 203 (1973) 17.
  - [18] D. Kessler, H. Mes, A. C. Thompson, H. L. Anderson, M. S. Dixit, C. K. Hargrove and R. J. McKee, Phys. Rev. C 11 (1975) 1719.
  - [19] J. M. Hu, *Nuclear Theory* Vol. I (Atomic Energy Press, Beijing, 1993) P281.
  - [20] P. Strasser *et al.*, Hyp. Int. 119 (1999) 317; Nucl. Instr. and Meth. A 460 (2001) 451; Eur. Phys. J. A 13 (2002) 197; Nucl. Phys. B (Proc. Suppl.) 149 (2005) 390.
  - [21] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28 (1956) 432; K. Alder and A. Winther, Coulomb Excitation (Academic Press, New York, 1966).
  - [22] H. L. Anderson *et al.*, Phys. Rev. 187 (1969) 1565.
  - [23] B. Fricke, Z. Phys. 218 (1969) 495.
  - [24] R. C. Barrett, S. J. Brodsky, G. W. Erlickson and M. H. Goldhaber, Phys. Rev. 166 (1968) 1589.